

# Laws of Induction

## LEAD-IN

1 Working in groups of three or four, state whether the following sentences are *true* or *false*, and correct the false ones.

- 1 In Shanghai, you can travel on a train that moves forward without touching the rails. It works thanks to electromagnetism. T F
- 2 Speedometers, which measure velocity in a car, are mechanical devices that don't work by electromagnetic induction. T F
- 3 The first electric guitar worked on the principle of direct electromagnetic induction. T F

2 Match the terms (1-6) to their corresponding definitions (a-f).

- |  |  |
|--|--|
| <input type="checkbox"/> 1 Scalar product        | a A flow of charge produced in a conductor by a time-varying magnetic flux.  |
| <input type="checkbox"/> 2 Magnetic flux         | b One of the two ends of a magnet.   |
| <input type="checkbox"/> 3 Induced current       | c They are defined in terms of a unit circle (i.e. a circle with a radius of 1).   |
| <input type="checkbox"/> 4 Magnetic pole         | d Work per unit of charge, equal to the electric potential difference produced across two open-circuit terminals.            |
| <input type="checkbox"/> 5 Goniometric functions | e A scalar quantity equal to the product of the magnitudes of two vectors and the cosine of the angle $\theta$ between them. |
| <input type="checkbox"/> 6 <i>emf</i>            | f The scalar product of the magnetic field vector $\vec{B}$ and the surface vector $\vec{A}$ .                               |

## READING AND LISTENING

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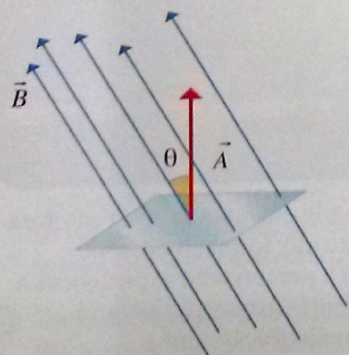
### Magnetic Flux

Faraday quantitatively investigated the mathematical relationship between induced current and the electromotive force (*emf*) which produces it. His experiments demonstrated that induced *emf* is proportional to the speed of change in magnetic field.

In addition, induced *emf* varies with loop area and the angle between the surface vector and the magnetic field lines. Since magnetic flux is the physical quantity that encompasses magnetic field, the given surface area, and their relative angle of inclination, Faraday's experiments suggest that *emf* is proportional to the change of the magnetic flux  $\Phi_B$  through a loop per unit time (i.e. rate of change).

Magnetic flux can be defined in a similar way to electric flux. So, when the magnetic field is uniform, it can be expressed as:

$$\Phi_B = BA \cos \theta \quad (8.1)$$

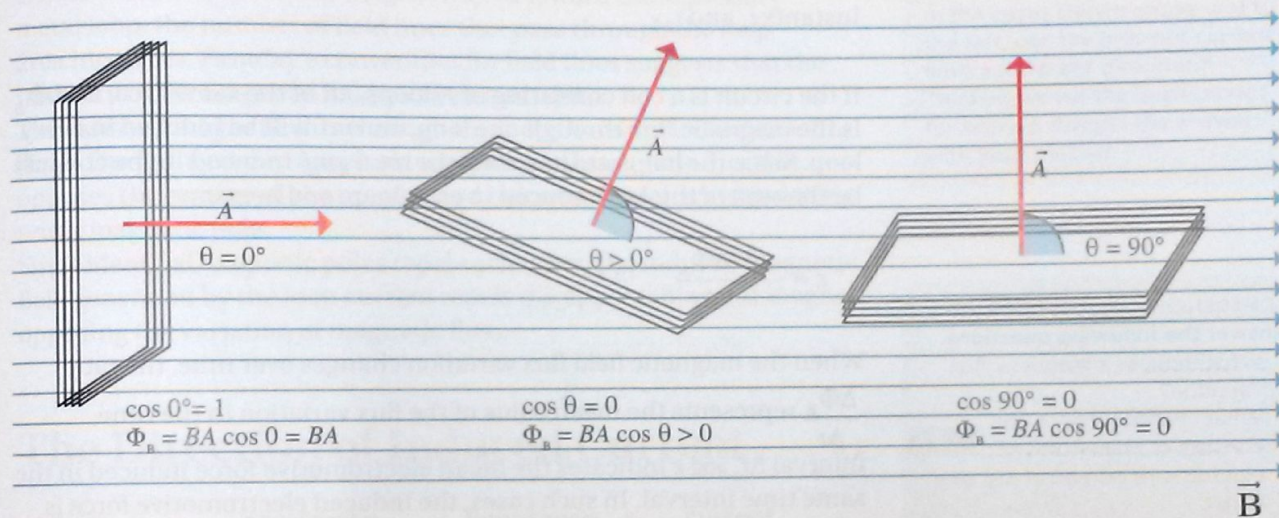




Magnetic flux  $\Phi_B$  is the scalar product of vectors  $\vec{B}$  and  $\vec{A}$ . The vector of an area is perpendicular to its surface, and its length is proportional to the amplitude of the area.

When the surface is perpendicular to the magnetic field  $\vec{B}$ , the angle  $\theta$  between  $\vec{B}$  and the vector  $\vec{A}$  is zero. So, since  $\cos 0^\circ$  is equal to 1,  $\Phi_B = B \cdot A \cdot \cos 0^\circ = B \cdot A$ . Finally, when the surface is parallel to  $\vec{B}$ ,  $\theta$  is equal to  $90^\circ$  and  $\cos 90^\circ = 0$ , so the magnetic flux is  $\Phi_B = 0$ .

Faraday postulated that **the number of field lines per unit area is proportional to the intensity of the field in that area**. This statement is also known as *Faraday's convention for field lines*, and is valid both for electric fields and magnetic fields. As a consequence, flux  $\Phi_B$  is also proportional to the number of lines passing through the loop's area. The following figure shows a loop and indicates the relationship between angle and intensity of magnetic flux.



According to Faraday's convention for field lines, when  $\theta = 90^\circ$ ,  $\Phi_B = 0$ , as no magnetic field lines pass through the loop's area. When the angle of inclination  $\theta$  increases, the value of  $\cos \theta$  increases, and magnetic flux increases as a result. Finally, when  $\theta = 0^\circ$  the maximum number of magnetic field lines pass through the surface, and so  $\Phi_B$  reaches its maximum value, which is equal to  $BA$ .

Since *tesla* is the unit of magnetic field, the unit of magnetic flux is the  $\text{tesla} \cdot \text{meter}^2$ , which can also be called a **weber (Wb)**.

$$1 \text{ Wb} = 1 \text{ T} \cdot \text{m}^2$$

To sum up, magnetic flux through an area varies as a result of:

- changes in the magnetic field intensity in that area;
- changes in the angle of inclination  $\theta$  between the magnetic field and the surface vector;
- changes in the area of the circuit loop.

After conducting many experimental tests, Faraday formed the following general law:

when the magnetic flux through an area changes, an *emf* is induced in the circuit, the intensity of which is proportional to the speed of flux variation.

#### COMPREHENSION QUESTIONS 1

Answer the following questions.

- 1 What are the parameters that influence the amplitude of induced current?
- 2 Is magnetic flux a vector quantity or a scalar one?
- 3 Which angle between vector  $\vec{B}$  and the surface area vector  $\vec{A}$  causes flux to be at its greatest, and which angle causes it to be zero?
- 4 What is Faraday's convention for field lines?
- 5 What unit is identical to a  $\text{tesla meter}^2$ ?
- 6 Would a variation in the size of the circuit area influence the magnetic flux across it?
- 7 What influences the intensity of induced *emf*?



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## Faraday-Neumann Law

Faraday's law states that when a magnetic flux variation occurs through a surface, an electromotive force is induced, and this induced *emf* is proportional to the rate of flux change.

The German physicist Franz Neumann (1798-1895) gave the mathematical form to Faraday's intuition, so the following law is often called the **Faraday-Neumann law of induction**:

$$\varepsilon = -\frac{\Delta\Phi_B}{\Delta t} \quad (8.2)$$

where  $\varepsilon$  indicates the electromotive force *emf* and  $\Delta\Phi_B = \Phi_B'' - \Phi_B'$  is the magnetic flux variation, which is equal to the difference between  $\Phi_B''$  and  $\Phi_B'$ , which are the respective magnetic field fluxes at the time instants  $t_2$  and  $t_1$ .  $\Delta t = t_2 - t_1$  is the time interval between the time instants  $t_1$  and  $t_2$ .

If the circuit is a coil consisting of  $N$  loops, all of the same area, and  $\Phi_B$  is the magnetic flux through one loop, an *emf* will be induced in every loop. Since the loops are in series, the total *emf* induced in the coil will be the sum of the *emf* induced in each loop, and hence:

$$\varepsilon = -N \cdot \frac{\Delta\Phi_B}{\Delta t} \quad (8.3)$$

### COMPREHENSION QUESTIONS 2

Answer the following questions.

- 1 What induces a magnetic flux variation?
- 2 What changes if each coil consists of many turns of electric wire instead of just one loop?
- 3 In what circumstances does

$\frac{\Delta\Phi_B}{\Delta t}$  evaluate the mean value of flux variation per unit of time?

When the magnetic field flux variation changes over time, the ratio

$\frac{\Delta\Phi_B}{\Delta t}$  represents the mean value of the flux variation in the time

interval  $\Delta t$ , and  $\varepsilon$  indicates the mean electromotive force induced in the same time interval. In such cases, the induced electromotive force is expressed by the following differential equation:

$$\varepsilon = -N \frac{d\Phi_B}{dt}$$

where the ratio  $\frac{d\Phi_B}{dt}$  is the derivative of magnetic flux with respect to time.

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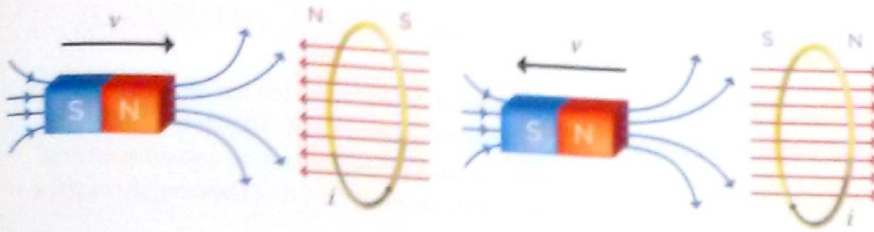
## Lenz's Law

The minus sign in the Faraday-Neumann equation indicates that the induced *emf* and the change in flux have opposite algebraic signs. This originates from experimental observation, but there is also a physical interpretation attributed to the German physicist Heinrich Lenz (1804-1865), known as **Lenz's law**:

when there is a variation in magnetic field flux through the area of a circuit loop, a current will be induced, and that current will generate a magnetic field flux which opposes the initial change in flow.

When you have read the above statement about Lenz's law, look at the following figures and captions.





**A** When a magnet with its north pole pointing rightward moves toward a motionless conducting loop, a current is induced in the direction shown. This current produces a leftward magnetic flux, counteracting the increasing external magnetic flux.

**B** When a magnet with its north pole pointing rightward moves away from a motionless conducting loop, a current is induced in the direction shown. This current produces a rightward magnetic flux, opposing the decreasing external magnetic flux.

Consider case A: as the bar magnet moves toward the stationary metal loop, the number of field lines that pass through the loop area increases. Faraday's convention for field lines suggests that the intensity of the magnetic field increases as well.

The external magnetic flux through the loop therefore increases with time. The induced current produces a leftward magnetic flux that opposes the motion of the magnet and behaves as a leftward-pointing north magnetic pole.

Since identical magnetic poles repel each other, the induced magnetic field generated by the loop current repels the approaching bar magnet, opposing the variation of magnetic flux.

In the cases shown above and to the left, can the induced current have a different direction? Thinking about the implications for energy, discuss the answer with your partner.

## The Direction of Induced Current

acquire	accelerate	hypothesis	current
anticlockwise	pole	result	
increasing	induced	bar magnet	

Considering the case described in figure A, assume as a working hypothesis that the direction of the induced current is <sup>1</sup>..... that the direction of the induced <sup>2</sup>..... is clockwise, such that the induced <sup>3</sup>..... magnetic field has the south <sup>4</sup>..... pole on the left side and therefore exerts an attraction force on the magnet <sup>5</sup>..... to the right. This force would cause the magnet to <sup>6</sup>....., thus <sup>7</sup>..... its velocity. This, in turn, would <sup>8</sup>..... in a rise in the induced current. Consequently, the force that produces the current would increase in magnitude, thereby increasing the current, and so on. The system would actually <sup>9</sup>..... energy without any additional input of energy. **This is clearly inconsistent with the law of conservation of energy.** So, in case A the current must be <sup>10</sup>.....

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#### LISTENING ACTIVITY

With your partner, consider the gaps in the text to the left. Try to put the right word in each gap. When you have finished, listen to the text to check your answers. Were they correct?

### Research Challenge

#### Perpetual motion

Over the centuries, many perpetual motion devices have been proposed and designed. What is a 'Brownian ratchet', and why doesn't it work?

#### COMPREHENSION QUESTIONS 3

Answer the following questions.

- 1 What does the minus sign in the Faraday-Neumann law equation indicate?
- 2 How does the direction of the induced current change if you move a bar magnet towards and away from a wire loop?
- 3 What is the scientific explanation for the direction of induced current?