N 300) BIS

$$\int\left(\sqrt{1-4x^2}+x\right)dx = \int\sqrt{1-4x^2} + \int x\,dx$$

$$= \frac{x^2}{2} + \int\sqrt{1-4x^2}\,dx$$

$$\int\sqrt{1-a^2x^2}\,dx \qquad 4x^2 = \text{sen}^2 t$$

$$2x = \text{sen}\, t \qquad x = \frac{1}{2}\,\text{sen}\, t$$

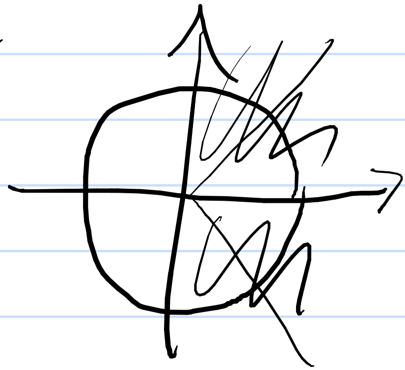
$$dx = \frac{1}{2}\cos t\, dt$$

$$\int\sqrt{1-4x^2}\,dx = \int\sqrt{1-\text{sen}^2 t}\cdot\frac{1}{2}\cos t\, dt =$$

$$= \frac{1}{2}\int|\cos t|\cos t\, dt$$

$1-4x^2 \geq 0 \qquad -\frac{1}{2} \leq x \leq \frac{1}{2}$

$-\frac{1}{2} \leq \frac{\sin t}{2} \leq \frac{1}{2}$



$-\frac{\pi}{2} < t < \frac{\pi}{2}$

$|\cos t| = \cos t$

$= \frac{1}{2}\int \cos^2 t \, dt = \frac{1}{2}\int \frac{\cos 2t + 1}{2} dt =$

$\cos 2\alpha = 2\cos^2\alpha - 1$

$\frac{\cos 2\alpha + 1}{2} = \cos^2\alpha$

$= \frac{1}{4}\int \cos 2t \, dt + \frac{1}{4}\int dt =$

$$= \frac{1}{4}\int \cos z \frac{1}{2}\,dz + \frac{1}{4}t =$$

$z = 2t$ $\qquad\qquad\qquad$ $\operatorname{sen} t = 2x$

$\frac{z}{2} = t$ $\qquad$ $\frac{dt}{dz} = \frac{1}{2}$ $\qquad$ $dt = \frac{1}{2}dz$

$$= \frac{1}{8}\operatorname{sen} 2t + \frac{1}{4}t + c$$

$t = \arcsin 2x$

$$\operatorname{sen} 2t = 2\operatorname{sen} t \cos t =$$

$= 2\operatorname{sen} t\sqrt{1-\operatorname{sen}^2 t}$ $\qquad$ $\operatorname{sen} t = 2x$

$$= 2\cdot 2x\cdot\sqrt{1-4x^2} = 4x\sqrt{1-4x^2}$$

$$\int\sqrt{1-4x^2}\,dx = \frac{x}{2}\sqrt{1-4x^2} + \frac{1}{4}\arcsin(2x) + c$$

$$\int \left(\sqrt{1-4x^2} + x\right) dx =$$

$$= \frac{x^2}{2} + \frac{x}{2}\sqrt{1-4x^2} + \frac{1}{4}\arcsin(2x) + C$$