FILA B

1) $y = \dfrac{x - \sqrt{x+6}}{4 - \sqrt{x^2-9}}$

Né pari né dispari

DOMINIO

$x+6 \geq 0 \qquad x \geq -6 \qquad$ A

$4-\sqrt{x^2-9} \neq 0$

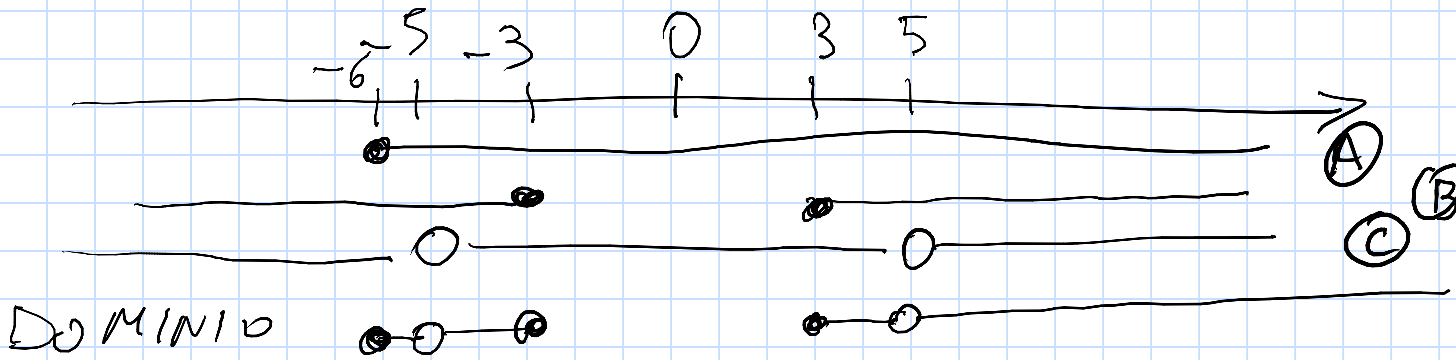
$x^2-9 \geq 0 \qquad x \leq -3 \vee x \geq +3 \qquad$ B

$4 \neq \sqrt{x^2-9} \qquad 16 \neq x^2-9$

$25 \neq x^2$

$x \neq \pm 5 \qquad$ C

Dominio $D \equiv [-6,5[\cup]5,3] \cup [3,5[\cup]5+\infty[$



INTERSEZIONI ASSI :

$x = 0$ non appartiene al dominio

$y = 0$ $\qquad 0 = x - \sqrt{x+6}$

$$\sqrt{x+6} = x \qquad a \geq \sqrt{b}$$

$$x + 6 = x^2 \qquad a^2 \geq$$

$$x^2 - x - 6 = 0$$

$$x - sx + p = 0$$

$$p = -6 \qquad \rightarrow x_1 = -2 \quad x_2 = 3$$
$$s = 1$$

$x_1 = -2$ NON APPARTIENE AL DOMINIO $\qquad A(3;0)$

POSITIVITÀ :

$$y \geq 0 \qquad \frac{x - \sqrt{x+6}}{4 - \sqrt{x^2-9}} \geq 0$$

$x - \sqrt{x+6} \geq 0 \rightarrow x \geq \sqrt{x+6}$ per $-6 < x < 0$

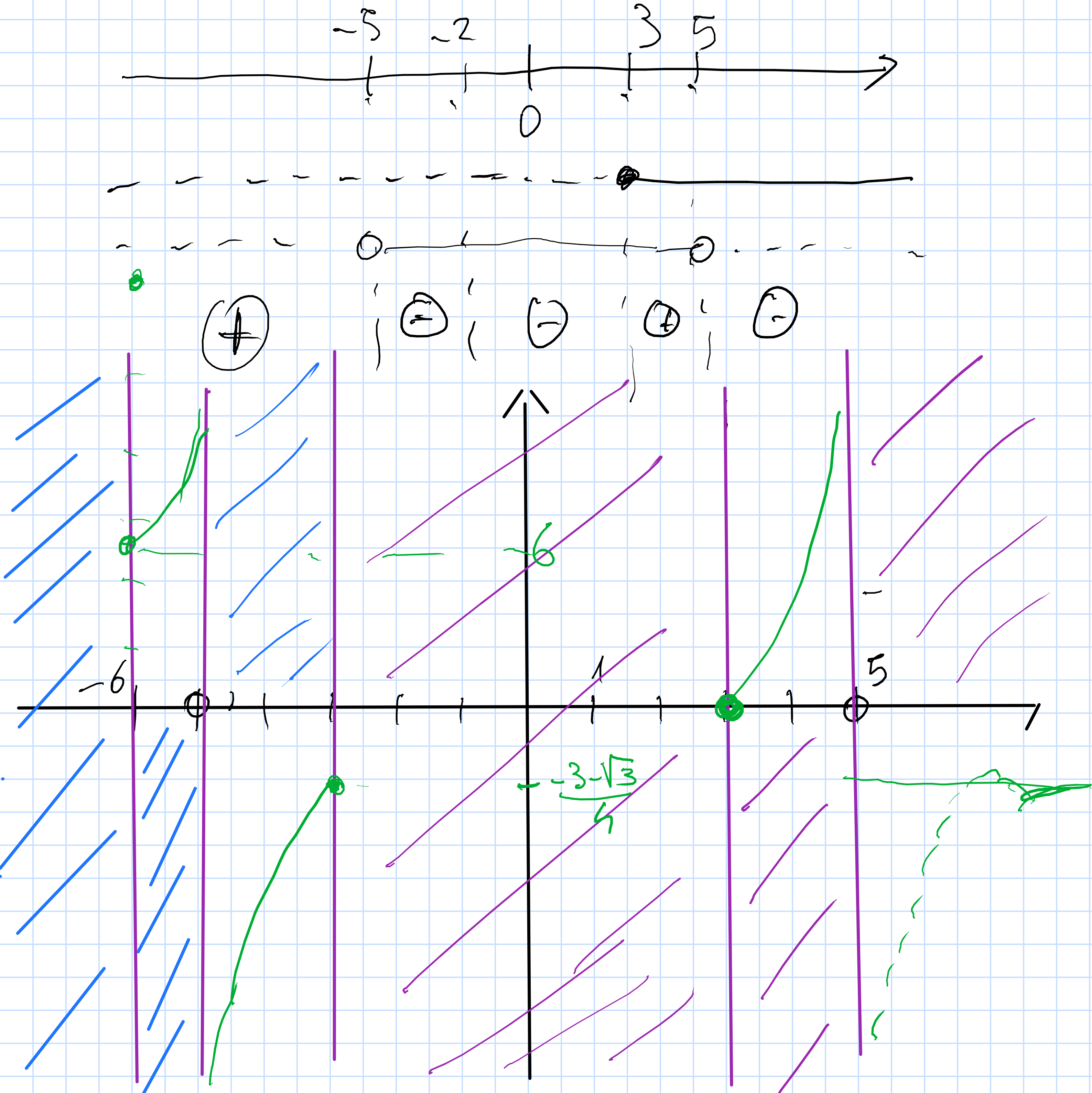
non ci sono soluz perchè la radice è positiva

Per $x \geq 0$ $\quad x^2 \geq x + 6 \quad x^2 - x - 6 \geq 0 \rightarrow x \geq 3$

$4 - \sqrt{x^2-9} > 0 \longrightarrow 4 > \sqrt{x^2-9}$ con $x^2 > 3$

$16 > x^2 - 9 \qquad 25 > x^2 \quad -5 < x < 5$

Ovviamente ristretti al DOMINIO



$$\lim_{x\to+\infty}\frac{x-\sqrt{x+6}}{4-\sqrt{x^2-9}} \qquad \frac{\infty}{\infty}$$

$$=\lim_{x\to+\infty}\frac{x\left(1-\sqrt{\frac{1}{x}+\frac{6}{x^2}}\right)}{|x|\left(\frac{4}{|x|}-\underbrace{\sqrt{1-\frac{9}{x^2}}}_{\to 1}\right)}=\lim_{x\to+\infty}\frac{x}{|x|}\,\frac{1}{-1}=-1$$

ASINTOTO ORIZZONTALE

$$\lim_{x \to 5^{\pm}} \frac{\overset{\nearrow 5}{x} - \overset{\nearrow \sqrt{11}}{\sqrt{x+6}}}{\underbrace{4 - \sqrt{x^2-9}}_{\searrow 0}} = \mp\infty$$

$$\lim_{x \to -5^{\pm}} f(x) = \pm\infty$$

$$\lim_{x \to -3^{-}} \frac{\overset{\nearrow -3}{x} - \overset{-\sqrt{3}}{\sqrt{x+6}}}{4 - \underset{\searrow 0^+}{\sqrt{x^2-9}}} = \frac{-3-\sqrt{3}}{4}$$

$$\lim_{x \to -6^{+}} \frac{x - \sqrt{x+6}}{4 - \sqrt{x^2-9}} = \lim_{x \to -6} \frac{-6-0}{4-\sqrt{25}} = 6$$

3) $\lim_{x \to +\infty} \frac{3x^2+2x-1}{-x^3+6x^2-7x+2} =$

$= \lim_{x \to +\infty} \frac{x^2\left(3+\frac{2}{x}-\frac{1}{x^2}\right)}{x^3\left(-1+\frac{6}{x}-\frac{7}{x^2}+\frac{2}{x^3}\right)} = 0^-$

4) $\lim_{x \to 4} \frac{\sqrt{x}-2}{x-4} = \lim_{x \to 4} \frac{\cancel{(\sqrt{x}-2)}}{\cancel{(\sqrt{x}-2)}(\sqrt{x}+2)} = \frac{1}{4}$

5)
$\lim_{x \to +\infty} \sqrt{x^2+x-2} - \sqrt{x^2-3x+4} =$

$= \lim_{x \to +\infty} \frac{x^2+x-2-(x^2-3x+4)}{\sqrt{x^2+x-2}+\sqrt{x^2-3x+4}} = \lim_{x \to +\infty} \frac{4x-6}{|x|\left(\sqrt{1+\frac{1}{x}-\frac{2}{x}}+\sqrt{1+\dots}\right)}$

$= \frac{4}{2} = 2$

6) $\lim\limits_{x \to 0^-} \dfrac{2^{\frac{1}{x}}}{\ln|x|} = \lim\limits_{x \to 0^-} \dfrac{2^{-\frac{1}{|x|}}}{\ln|x|} =$

$= \lim\limits_{x \to 0^-} \dfrac{\left.\left(\frac{1}{2}\right)^{\frac{1}{|x|}}\right\} \to 0}{\underbrace{\ln|x|}_{\to -\infty}} = 0^-$

7) $\lim\limits_{x \to +\infty} \dfrac{e^{-x^2}}{\ln x} = \dfrac{0}{\infty} = 0$

8) $\lim\limits_{x \to 0} \dfrac{e^{2x}-1}{\sin 5x} = \lim\limits_{x \to 0} \dfrac{2}{5} \dfrac{e^{2x}-1}{2x} \cdot \dfrac{5x}{\sin 5x} =$

$= \dfrac{2}{5}$

9) $\lim\limits_{x \to +\infty} \dfrac{2x + \sin x}{7x + \cos x}$ $\qquad \dfrac{\infty}{\infty}$

$= \lim\limits_{x \to +\infty} \dfrac{x\left(2 + \overbrace{\frac{\sin x}{x}}^{\to 0}\right)}{x\left(7 + \underbrace{\frac{\cos x}{x}}_{\to 0}\right)} = \dfrac{2}{7}$

10) $$\lim_{x\to\infty}\left(\frac{x+3}{x+1}\right)^x = \lim_{x\to\infty}\left[\frac{x\left(1+\frac{3}{x}\right)}{x\left(1+\frac{1}{x}\right)}\right]^x =$$

$$= \lim_{x\to\infty}\frac{\left(1+\frac{3}{x}\right)^x}{\left(1+\frac{1}{x}\right)^x} = \lim_{x\to\infty}\frac{\left(1+\frac{3}{x}\right)^x}{e} =$$

$$y=\frac{x}{3} \qquad x=3y$$

$$= \lim_{y\to\infty}\frac{1}{e}\left(1+\frac{1}{y}\right)^{3y} = \frac{1}{e}\left[\lim_{y\to\infty}\left(1+\frac{1}{y}\right)^y\right]^3 = \frac{1}{e}\cdot e^3 = e^2$$

11) $$\lim_{x\to 3}\frac{x-3}{\sqrt{x+6}-3} \qquad \frac{0}{0}$$

$$= \lim_{x\to 3}\frac{(x-3)(\sqrt{x+6}+3)}{x+6-9} = \lim_{x\to 3}\frac{\cancel{(x-3)}(\sqrt{x+6}+3)}{\cancel{x-3}} =$$

$$= 6$$

12) $\lim_{x\to\infty}\left(\frac{5x-3}{5x-1}\right)^{-2x}=\lim_{x\to\infty}\left(\frac{5x-1}{5x-3}\right)^{2x}=$

$=\lim_{x\to\infty}\left(\frac{5x-3+2}{5x-3}\right)^{2x}=\lim_{x\to\infty}\left(1+\frac{2}{5x-3}\right)^{2x}$

$\frac{2}{5x-3}=\frac{1}{y}$ $\quad 2y=5x-3$ $\quad \frac{2y+3}{5}=x$

$x\to\infty$ $\quad y\to\infty$

$=\lim_{y\to\infty}\left(1+\frac{1}{y}\right)^{\frac{2}{5}(2y+3)}=\lim_{y\to\infty}\left[\left(1+\frac{1}{y}\right)^{y}\right]^{\frac{4}{5}}\cdot\left(1+\frac{1}{y}\right)^{\frac{6}{5}}=$

$=e^{\frac{4}{5}}$