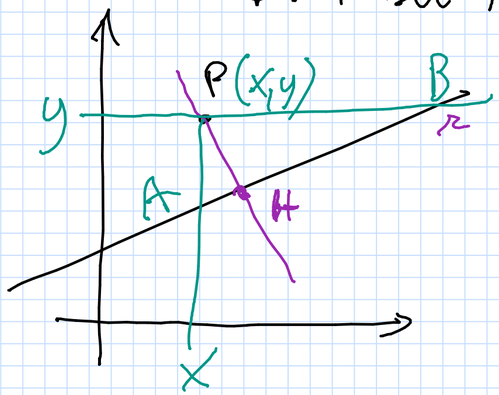
CALCOLO DISTANZA PUNTO RETTA:

SIA H LA PROIEZIONE ORTOGONALE DI P SU r



$P(4,10)$

$r: y = 3 + \frac{1}{2}x$

$y = -2x + q$

$10 = -8 + q$

$q = 18$

$\begin{cases} y = -2x + 18 \\ y = \frac{1}{2}x + 3 \end{cases}$

$0 = -2x - \frac{1}{2}x + 18 - 3$

$\frac{5}{2}x = 15 \qquad x = \frac{2}{5} \cdot 15^{3} = 6$

$y = -12 + 18 = 6$

$H(6,6)$

1) retta perp a r per P

2) intersezione tra le rette

3) distanza tra P e intersezione

$d = \sqrt{(6-4)^2 + (6-10)^2} =$

$= \sqrt{4 + 16} = \sqrt{20} = 2\sqrt{5}$

$\neq \sqrt{4} + \sqrt{16}$

$= \sqrt{2^2 \cdot 5} = \sqrt{2^2} \cdot \sqrt{5} = 2\sqrt{5}$

$\sqrt{3} \cdot \sqrt{3} = 3$

$\left(\sqrt{3}\right)^2 = 3$

$3^{\frac{1}{2}} \cdot 3^{\frac{1}{2}} = 3^{\frac{1}{2}+\frac{1}{2}} = 3^{1} = 3$

$\left(3^{\frac{1}{2}}\right)^2 = 3^{1} = 3$

$\longrightarrow \quad 3^{\frac{1}{2}} = \sqrt{3}$

$\sqrt[3]{3} = 3^{\frac{1}{3}}$

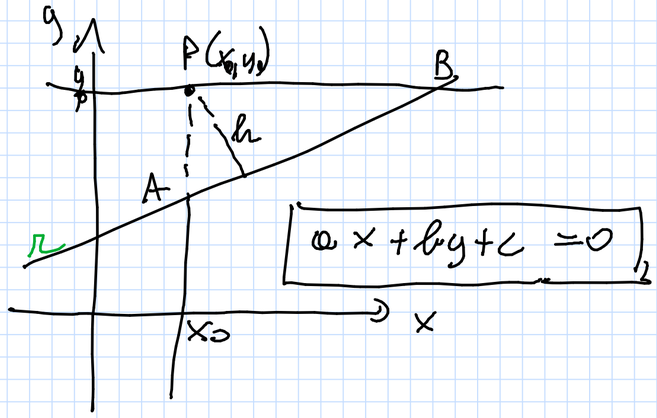
$\sqrt[3]{3} = 3$

$\left(3^{\frac{1}{4}}\right)^3 = 3^{\frac{3}{4}}$

$\left(3^{\frac{1}{3}}\right)^3 = 3^{1} = 3$

DIMOSTRAZIONE FORMULA DISTANZA PUNTO RETTA

(ENRICO MATTEI)



APB è rettangolo

$$Q = \frac{AP \cdot PB}{2} = \frac{AB \cdot h}{2}$$

$$\boxed{h = \frac{AP \cdot PB}{AB}}$$

$P(x_0, y_0)$

coordinate di $A\left(x_0 ; \frac{-ax_0 - c}{b}\right)$

$ax_0 + by_A + c = 0 \rightarrow by_A = -ax_0 - c \quad y_A = \frac{-ax_0 - c}{b}$

coordinate di $B\left(\frac{-c - by_0}{a} ; y_0\right)$

$ax_B + by_0 + c = 0 \rightarrow ax_B = -c - by_0 \rightarrow x_B = \frac{-c - by_0}{a}$

$$AP = \sqrt{(x_A - x_P)^2 + (y_A - y_P)^2} = \sqrt{(x_0 - x_0)^2 + \left(-\frac{ax_0 - c}{b} - y_0\right)^2} =$$

$$= \sqrt{\left(\frac{-ax_0 - c - by_0}{b}\right)^2} = \sqrt{\left(\frac{-ax_0 + by_0 + c}{b}\right)^2} = \left|\frac{ax_0 + by_0 + c}{b}\right| \leq$$

$$PB = \sqrt{(x_P - x_B)^2 + (y_P - y_B)^2} = \sqrt{\left(x_0 - \frac{-c - by_0}{a}\right)^2 + (y_0 - y_0)^2} =$$

$$= \sqrt{\left(\frac{ax_0 + c + by_0}{a}\right)^2} = \left|\frac{ax_0 + by_0 + c}{a}\right| \leq$$

$$AB = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2} = \sqrt{\left(x_0 - \frac{-c - by_0}{a}\right)^2 + \left(\frac{-ax_0 - c}{b} - y_0\right)^2} =$$

$$= \sqrt{\left(\frac{ax_0 + by_0 + c}{a}\right)^2 + \left(\frac{ax_0 + by_0 + c}{b}\right)^2} = \sqrt{\frac{(ax_0 + by_0 + c)^2}{a^2} + \frac{(ax_0 + by_0 + c)^2}{b^2}} =$$

$$= \sqrt{\frac{b^2(ax_0 + by_0 + c)^2 + (ax_0 + by_0 + c)^2 a^2}{a^2 b^2}} = \sqrt{\frac{(ax_0 + by_0 + c)^2 (b^2 + a^2)}{a^2 b^2}}$$

$$= \frac{|ax_0 + by_0 + c|}{|ab|} \cdot \sqrt{a^2 + b^2}$$

$$h = \frac{AP \cdot BP}{AB} = \left|\frac{ax_0 + by_0 + c}{b}\right| \cdot \left|\frac{ax_0 + by_0 + c}{a}\right| \cdot \frac{|ab|}{|ax_0 + by_0 + c|} \cdot \frac{1}{\sqrt{a^2 + b^2}}$$

$$\boxed{h = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}}$$

$P(x_0, y_0)$

$ax + by + c = 0$

$\sqrt{(-3)^2} = 3$

$|-3|$

ESEMPIO ESERCIZIO:

retta: $3x + 2y - 1 = 0$

punto $P(-1, 5)$

$ax + by + c = 0$

$a = 3 \quad b = 2 \quad c = -1$

$x_0 = -1 \quad y_0 = 5$

distanza: $d = \dfrac{|3(-1) + 2(5) - 1|}{\sqrt{3^2 + 2^2}} = \dfrac{|-3 + 10 - 1|}{\sqrt{13}} = \dfrac{6}{\sqrt{13}}$